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$\therefore\ 300 : 6 = 300\frac{1}{2} : 6\frac{1}{100}$,

$300 : 6 = 301\frac{1}{2} : 6\frac{3}{100}$,

$300 : 6 = 302\frac{1}{2} : 6\frac{5}{100}$.

. . . .

Let n=number of turns.

$\therefore\ 2\pi(6\frac{1}{100}+6\frac{3}{100}+6\frac{5}{100}+\text{to } n \text{ terms})=1200$ inches.

$\therefore\ 2\pi(6+6+6+\ldots\ldots)+2\pi(\frac{1}{100}+\frac{3}{100}+\frac{5}{100}+\ldots\ldots)=1200$.

$\therefore\ 12\pi n+\pi n^2/1200=100$. $\therefore\ n=31.03$ coils. From the previous problem,

$$s=\frac{4\pi rm}{l^2}\int_{25}^{27\frac{7}{12}}\sqrt{1+\frac{\pi^2m^2}{l^2}(l-x)^2}.xdx, \text{ where } m=331,$$

the number of coils of rope to top of cone beginning with the bottom coil, and $l=331$ inches the height$=27\frac{7}{12}$ feet; also let 25 feet$=l'$, $r=\frac{331}{600}$ feet=radius of base of cone at bottom of rope.

$$\therefore\ s=\frac{4\pi rm}{l^2}\int_{l'}^{l}\sqrt{1+\frac{\pi^2m^2}{l^2}(l-x)^2}.xdx.$$

$$\therefore\ s=\frac{4r}{3\pi m}+\frac{2r}{3l^3\pi m}[(l^2+ll'-2l'^2)\pi^2m^2-2l^2]\sqrt{l^2+(l-l')^2\pi^2m^2}$$

$$+2r\log\left(\frac{(l-l')\pi m+\sqrt{l^2+(l-l')^2\pi^2m^2}}{l}\right).$$

$$s=(1/450\pi)+(1/900\pi)(28861\pi^2-2)\sqrt{1+961\pi^2}+\tfrac{331}{300}\log(31\pi+\sqrt{1+961\pi^2}).$$

$s=9817.69235$ feet.

In this solution the rope is unwound from the bottom.

MECHANICS.

55. Proposed by **ALFRED HUME, C. E., D. Sc.**, Professor of Mathematics, University of Mississippi, University P. O., Miss.

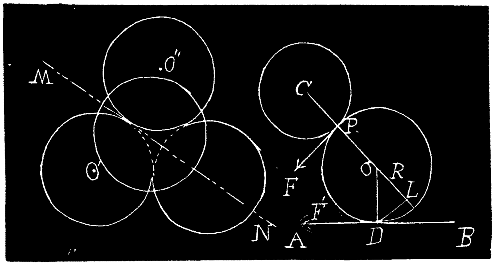
Three equal heavy spheres, each of weight W, are placed on a rough ground just not touching each other. A fourth sphere of weight nW is placed on the top touching all three. Show that there is equilibrium if the coefficient of friction between two spheres is greater than $\tan\frac{1}{2}\alpha$, and that between a sphere and the ground is greater than $n\tan\frac{1}{2}\alpha/(n+3)$, where α is the inclination to the vertical of the straight line joining the centers of the upper and one lower sphere.

Solution by the PROPOSER.

I.

In what different ways may motion occur, if the friction be such as to permit motion?

So long as the lower spheres maintain their positions, the upper will not move.



PLAN. SECTION MN.

How, then, may the lower move? Referring to the section—

1. By sliding in the direction *BA*;
2. By rolling in the direction *BA*;
3. By rolling in the direction *AB*;
4. By sliding in the direction *AB*.

Motions (1) and (2) cannot take place because, even if this did not at once bring the lower spheres into contact, such would necessarily raise the upper sphere and, evidently, there are no forces acting which could have this effect. Having disposed of (1) and (2), consider (3).

The forces acting on the sphere whose center is *O* are its own weight, the friction F', the friction F, and the normal action between the two spheres, R. Rolling in the direction *AB* will not occur unless the moment of R about D is greater than that of F. Equating these two moments,

$$F.PL = R.DL,$$

$$F = (DL/PL)R$$

$$= \tan\tfrac{1}{2}\alpha . R,$$

since $\angle DPL = \frac{1}{2}\angle DOL$, and $\angle DOL = \alpha$, OD being vertical and OL the prolongation of CO.

If μ=coefficient of friction between the two spheres, $F = \mu R$.

$\therefore\ \mu = \tan\frac{1}{2}\alpha$.

If, then, the coefficient of friction between the spheres whose centers are C and O is greater than $\tan\frac{1}{2}\alpha$, F will prevent motion (3).

Now consider (4).

This will take place unless F' is equal to, or greater than, $R\sin\alpha - F\cos\alpha$.

The normal pressure on the ground at D is the weight of sphere O plus one-third of the weight of sphere C, or $W + \frac{1}{3}nW$.

If μ' is the coefficient of friction between the sphere and the ground, $F' = \mu'(n+3)\frac{1}{3}W$, and, we have, for equilibrium,

$$\mu'(n+3)\tfrac{1}{3}W = R\sin\alpha - F\cos\alpha$$

$$= R(\sin\alpha - \tan\tfrac{1}{2}\alpha\cos\alpha),$$

taking the least value of F consistent with equilibrium. From this,

$$R = \frac{\mu'(n+3)}{\tan\frac{1}{2}\alpha} . \tfrac{1}{3}W.$$

Consider the forces acting on the upper sphere—its own weight nW, F, and R reversed in direction, and, in addition, the actions F and R from each of the two other spheres, O' and O'' (see plan). Resolving vertically,

$$nW=3R\cos\alpha+3F\sin\alpha$$
$$=3R(\cos\alpha+\tan\tfrac{1}{2}\alpha\sin\alpha)=3R;$$
$$R=\tfrac{1}{3}nW.$$

Equating the two values of R and solving for μ',

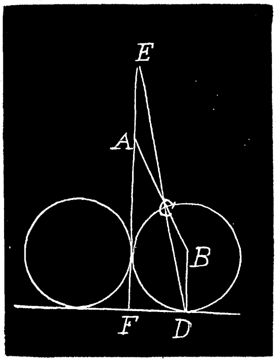
$$\mu'=\tan\tfrac{1}{2}\alpha.\frac{n}{n+3}.$$

REMARKS. Looking at the section it might seem, at a glance, that R ought to bs $\frac{1}{3}nW\cos\alpha$, whereas it has been found to be $\frac{1}{3}nW$. It must be remembered that the two spheres, C and O, are not in equilibrium by themselves, C being held in position by reactions and friction from O' and O'' as well as from O.

It may be noted, too, that the friction brought into play at P may not be sufficient to prevent sliding. In order that motion of this kind may not take place the coefficient of friction must not be less than $\tan\alpha$. But the reason why C does not slide is that it cannot descend unless O rolls or slides out of its way toward B. And, as has been seen, neither of these motions can occur if $\mu>\tan\frac{1}{2}\alpha$ and $\mu'>\tan\frac{1}{2}\alpha.n/(n+3)$.

III. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va.

Let W=weight of one of the equal spheres, W_1=weight of upper sphere, $\angle FAB=\alpha$.



Since sphere B and reaction of the ground acts through D, the total resistance between spheres A and B must act through D.

Hence the total resistance between A and B acts in the line ECD.

$\therefore$ The angle of friction between A and B must be $\overset{=}{>}$ than $\angle ACE=\frac{1}{2}\alpha$, $\therefore$ coefficient of friction $\overset{=}{>}\tan\frac{1}{2}\alpha$.

Let μ=coefficient of friction between B and the ground, R=total normal, P=total tangential resistance at D.

$$\therefore \mu R=P \text{ or } \mu=P/R.$$

$$\therefore \mu=(\tfrac{1}{3}W_1\sin\tfrac{1}{2}\alpha\cos\tfrac{1}{2}\alpha)/[(\tfrac{1}{3}W_1+W)\cos^2\tfrac{1}{2}\alpha].$$

$$\therefore \mu=\frac{W_1}{W_1+3W}\tan\tfrac{1}{2}\alpha=\frac{n}{n+3}\tan\tfrac{1}{2}\alpha, \text{ when } W_1=nW.$$

Let $AC=b$, $CB=c$. Then $\sin\alpha=\dfrac{2c}{\sqrt{3}(b+c)}$.

$$\therefore \tan\tfrac{1}{2}\alpha=\frac{(b+c)\sqrt{3}-\sqrt{3b^2+6bc-c^2}}{2c}.$$

$$\therefore \tan\tfrac{1}{2}\alpha=\sqrt{3}-\sqrt{2} \text{ when } b=c.$$